Physics 222, Fall 1996
Solutions for Homework Set #19
Serway, Chapter 29

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Question 9

Problem: Suppose a photograph were made of a person's face using only a few photons. Would the result be simply a faint image of the face? Discuss.

Solution: Images as recognized by the human eye are interference patterns which are made up from light that is reflected from different surfaces. If we see the face of a person this is an interference pattern and so is a photograph. Taking only a few photons would not give a faint picture of the face. You would rather see single photons than a faint picture. However, if you wait for a long time and collect more photons the image will begin to form and you will recognize the interference pattern you are used to. This will be a faint image of the face. If you wait even longer the contrast will be more and more enhanced until you get a real photo. This is very similar to the situation shown in Fig. 29.13 where this experiment was used to demonstrate the wave character of electrons. In the same manner one can show the particle character of light.

Problem 5

Problem: What is the peak wavelength emitted by the human body? Assume a body temperature of 98.6° F and use the Wien displacement law. In what part of the electromagnetic spectrum does this wavelength lie?

Solution: The body temperature is $T = 98.6^\circ\ \mathrm{F} = 310.15\ \mathrm{K}$. With the Wien displacement law (Eq. [29.1]) we get

$$l_{\max} = \frac{0.2898\times 10^{-2}\ \mathrm{m}\cdot\mathrm{K}}{310.15\ \mathrm{K}} = 9.34\times 10^{-6}\ \mathrm{m} = 9.34\ m\mathrm{m}$$

The radiation of the human body is in the infrared region of the spectrum far beyond what is visible for the human eye.

Problem 13

Problem: From the scattering of sunlight, Thomson calculated the classical radius of the electron having a value of $2.82\times 10^{-15}\ \mathrm{m}$. If sunlight with an intensity of 500 W/m^2 falls on a disk with this radius, estimate the time required to accumulate 1.0 eV of energy. Assume that light is a classical wave and that the light is striking the disk is completely absorbed. How does your estimate compare with the observation that photoelectrons are promptly (within 10^{-9} s) emitted?

Solution: We know the classical radius of the electron R_e and the power density of the sunlight P_A striking the disk with area $A = pR_e^2$. The power absorbed by the disk is $P = P_A \times pR_e^2$. The time needed to accumulate an energy E is given by $t = E/P$. Now we can plug in what we know (do not forget to convert between J and eV):

$$t = \frac{1 \text{ eV}}{500 \frac{\text{W}}{\text{m}^2} \times p \times (2.82 \times 10^{-15} \text{ m})^2} = 1.28 \times 10^7 \text{ s} = 148 \text{ d}$$

This is much longer than the time observed in an experiment which is about 10^{-9} s. The solution is the quantization of light. Light-quanta are called photons and each photon carries energy $E = hf$. When sunlight hits a metallic surface some of the electrons will absorb the incoming radiation. If the energy of the photons is sufficiently high one will measure photoelectrons. This certainly contadicts the classical idea of absorption and cannot be understood in the classical approach. The photoelectric effect is an important example for an experiment where the particle character of light (photons) is needed to explain the observed results.

Problem 18

Problem: Light of wavelength 300 nm is incident on a metallic surface. If the stopping potential for the photoelectric effect is 1.2 V, find (a) the maximum energy of the emitted electrons, (b) the work function, and (c) the cutoff wavelength.

Solution: (a) An electron has energy E and loses all its (kinetic) energy moving against the stopping potential of 1.2 V. By definition of the unit eV (when one electron passes a potential difference of 1 V it has the energy 1 eV) the potential difference of 1.2 V corresponds to an initial kinetic energy of $1.2 \text{ eV} = 1.92 \times 10^{-19} \text{ J}$. Another way to get this result is by explicitly calculating $K_{MAX} = qV_S$ (Eq. [29.4]).

(b) Since for photons we have $c = lf$ we can express the energy in terms of wavelength $E = hf = \frac{hc}{l}$. Use this in Eq. [29.5] and solve for the work function Φ:

$$\frac{(6.63 \times 10^{-34} \text{ Js})(3 \times 10^8 \text{ m/s})}{300 \times 10^{-9} \text{ m}} = \Phi + 1.92 \times 10^{-19} \text{ J} \Rightarrow \Phi = 4.71 \times 10^{-19} \text{ J} = 2.94 \text{ eV}$$

(c) The cutoff wavelength is given by Eq. [29.6]:

$$l_C = \frac{hc}{\Phi} = \frac{(6.63 \times 10^{-34} \text{ Js})(3 \times 10^8 \text{ m/s})}{4.71 \times 10^{-19} \text{ J}} = 422 \text{ nm}.$$

Problem 21

Problem: A gamma-ray photon with an energy equal to the rest energy of an electron (511 keV) collides with an electron that is initially at rest. Calculate the kinteic energy aquired by the electron if the photon is scattered 30° from its original line of approach.

Solution: The photon comes in with an energy of 511 keV and is scattered by an electron of mass m_e. The scattering angle is $f = 30^\circ$. With Eq. [29.7] one can calculate the change in wavelength for the scattered photon.

$$\Delta l = \frac{h}{m_e c}(1 - \cos f) = \frac{\left(6.626\times10^{-34}\ \text{Js}\right)}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(3\times10^{8}\ \text{m/s}\right)}\left(1-\cos 30^\circ\right) = 3.25\times10^{-13}\ \text{m}$$

The incoming photon has a wavelength $l_{in} = 2.426\times10^{-12}\ \text{m}$ whereas the outgoing photon has a wavelength $l_{out} = l_{in} + \Delta l = 2.75\times10^{-12}\ \text{m}$. From this we can determine the energy of the outgoing photon to E_{out} = 450 keV. Since the incoming had an energy of 511 keV and energy is conserved the difference of 60.4 keV must be transferred into kinetic energy of the electron.

Problem 31

Problem: The distance between adjacent atoms in crystals is in the order of $1\,\text{Å}$. The use of electrons in diffraction studies of crystals requires that the de Broglie wavelength of the electrons be on the order of the distance between atoms in the crystals. What must be the minimum energy (in electron volts) of electrons to be used for this purpose?

Solution: The de Broglie wavelength of the electrons must be of the order of 1 Å = 0.1 nm. Since the de Broglie wavelength is given by Eq. [29.9] we can solve for the velocity:

$$l = \frac{h}{mv} = 0.1\ \text{nm} \Rightarrow v = \frac{h}{ml}$$

When we now calculate the kinetic energy of the electrons we use the velocity from the de Broglie relation:

$$E_{KIN} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{h}{ml}\right)^2 = \frac{1}{2}\frac{h^2}{ml^2} = \frac{1}{2}\frac{\left(6.626\times10^{-34}\ \text{Js}\right)^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.1\times10^{-9}\ \text{m}\right)^2} = 2.41\times10^{-17}\ \text{J} = 150\ \text{eV}$$